Applicability of two hybrid sound prediction methods for assessing in-duct sound absorbers of turbocharger compressors

Clemens Freidhager¹,*, Stefan Schoder¹, Paul Maurerlehner¹, Andreas Renz², Stefan Becker², and Manfred Kaltenbacher¹

¹Graz University of Technology, IGTE, Inffeldgasse 18, 8010 Graz, Austria
²Friedrich-Alexander Universität Erlangen-Nürnberg, Cauerstraße 4, 91058 Erlangen, Germany

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Abstract — We analyze the differences between the Ffowcs-Williams and Hawking’s approach and a new sound propagation approach based on the finite element method used for solving Lighthill’s aeroacoustic wave equation for compressible flows. In addition, we discuss the applicability of both methods. The sound propagation approach based on Lighthill’s equation introduces a flow-interface boundary condition, similar to permeable boundaries in the Ffowcs–Williams and Hawking’s analogy, which allows the omission of complex geometries in propagation domains. This enables to reduce numeric effort and storage requirements. Thereby, the hybrid aeroacoustic workflow is considered, for which aeroacoustic source terms are computed to couple a flow and a separated acoustic propagation simulation. We present an extensive investigation of Lighthill’s source terms in the sense of the proposed weak formulation of Lighthill’s equation. For validation, measurements from a cold gas test rig are used. In addition, the possibilities of applying both sound propagation methods for investigating the influence of resonators and sound absorbers are discussed.

Keywords: Aeroacoustic, Lighthill, Ffowcs Williams-Hawking

1 Introduction

One of the primary sources of noise pollution are technical devices [1]. The noise produced by technical applications is mainly connected to transportation, most often airplanes and highway vehicles [2]. As a result, in 2014, the European Commission introduced the regulation (EU) 540/2014 [3] for reducing noise in the automotive industry by 25% until the last implementation phase starting in 2024. As reference noise of the automotive industry, the year 2014 was considered. This is in agreement with the 17 sustainable development goals established by the United Nations [4]. Consequently, noise from car power units has recently become of great interest for industry-related science.

A common approach for investigating aeroacoustic phenomena is using simulations. However, because of the disparity of scales, current computational limitations do not allow direct simulations of flow and acoustics in complex technical applications [5, 6]. As a result, a general approach in literature is performing flow simulations using a compressible Detached Eddy Simulation (DES), in most cases with an underlying k-ω shear stress transport (SST) turbulence model [7–14], or a compressible Large Eddy Simulation (LES) [15–19]. Performing compressible flow simulations has two major disadvantages. Firstly, flow simulation solvers are affected by numeric dissipation [17, 20], which becomes problematic for investigating acoustic propagation. The phenomenon of numerical dissipation is particularly problematic for low-order solvers and can be mitigated by finer discretization [20]. However, this significantly increases the numerical effort. Secondly, performing flow simulations of complex applications is computationally expensive. Depending on the available computational resources, computing enhanced flow simulations takes a lot of time, making it impractical to design and assess resonators and sound absorbers, which are common to reduce noise in ducted systems. This paper will discuss two different approaches to overcome these limitations. Firstly, by applying the integral Ffowcs Williams and Hawking’s (FW–H) analogy [17, 21], restrictions due to numeric dissipation do not affect simulation results. However, for in-duct simulations using the FW–H analogy, the limitations arising from the used free-field Green’s function have to be taken into account. The second approach for coping with the restrictions of flow solvers is a novel Lighthill propagation approach (LH-FE) for compressible flows based on the Finite Element (FE) method. LH-FE allows quickly incorporating different resonator and sound absorbers geometries [22]. Various works have already dealt with LH-FE in incompressible flows [23–25]. Furthermore, in 2005, Caro et al. [26] applied
Lighthill’s equation in combination with the FE method to an incompressible flow, to a pseudo compressible flow, where minor density variations are accounted for in the continuity equation, and to an isentropic flow, where the density is only a function of the pressure. However, none of those three cases account for the highly compressible flow with local Mach numbers $Ma > 1$ as in a turbocharger compressor. Consequently, we present an LH-FE approach coping with compressible flows. In addition, we investigate both the LH-FE approach and the FW–H approach by applying them to a turbocharger compressor. Furthermore, for validation of simulation results, cold gas test rig measurements were used. Finally, the applicability of the LH-FE approach for optimizing duct systems acoustically is shown by investigating the influence of a resonator insert.

This research article is structured as follows: Section 2 provides explanations of the considered FW–H approach (see Sect. 2.1) and the LH-FE approach (see Sect. 2.2). In Section 3, the flow and propagation setups are discussed. In Section 4, an extensive investigation of Lighthill’s source terms is performed. In addition, flow and propagation simulation results are investigated and validated with measurements. Furthermore, in Section 4.3, general applicability and practical aspects of the LH-FE approach are discussed. Finally, the presented work is concluded in Section 5.

2 Methodology

Subsequently, the FW–H analogy and the LH-FE are discussed. Both sound propagation approaches are based on compressible flow simulations, which have to be performed beforehand. Furthermore, both approaches rest on Lighthill’s inhomogeneous wave equation [27, 28] which reads as

$$
\left( \frac{1}{c_0^2} \frac{\partial^2}{\partial t^2} - \Delta \right) (c_0^2 \rho') = \nabla \cdot \nabla \cdot [L].
$$

(1)

Thereby, $c_0$ denotes the ambient speed of sound, $\Delta$ the Laplace operator, $\rho' = \rho - \rho_0$ the density fluctuations, and $\nabla$ the Nabla operator. Lighthill’s equation directly reformulates the compressible flow equations without any simplifications. Lighthill’s stress tensor $[L]$ can be separated into three main parts, (1) a part due to Reynolds stresses, (2) an excess part, and (3) a part due to viscous stresses

$$
[L] = \frac{\rho \mathbf{v} \otimes \mathbf{v}}{Re} + (\rho' - c_0^2 \rho') [I] - \frac{\mathbf{r}}{\ret}.
$$

(2)

For low Mach number flows, the principal sound generators are the fluctuating Reynolds stresses. Consequently for $Ma < 0.3$ the Reynolds stress term becomes dominant [27, 29]. Furthermore, the excess term describes density deviation in the source flow with respect to the mean flow. As a result, the excess term becomes dominant for high Mach number “Ma” flows, as well as for thermoacoustics [29, 30] and combustion noise [31]. The viscous stress term, however, is only contributing substantially for small Reynolds numbers “Re” [32]. Consequently, the viscous stress term is neglected for most technical applications, as shown in Section 4.1.

2.1 Ffowcs-Williams and Hawking’s Analogy

Currently, the FW–H’s analogy [21] is presumably one of the most used analogies for computing far-field pressure propagation of moving geometries [17, 33]. Considering $[P] = (p - p_0) [I]$ as the differences of the pressure from its mean value $p_0 [I]$ (with $[I]$ being the unit tensor), and $\mathbf{v}_T$ as surface velocity, the integral representation from the FW–H formulation can be expressed as [21, 34]

$$
\rho' = \frac{1}{4 \pi c_0^2} \nabla \cdot \nabla \cdot \int_{\Omega} \left[ \frac{[L]}{r[1 - Ma]} \right]_{\ret} d\Omega
$$

$$
- \frac{1}{4 \pi c_0^2} \nabla \cdot \int_{\Gamma} \left[ \frac{[P] \cdot \mathbf{n}}{r[1 - Ma]} \right]_{\ret} d\Gamma
$$

$$
+ \frac{1}{4 \pi c_0^2} \frac{\partial}{\partial t} \int_{\Gamma} \left[ \frac{p_0 \mathbf{v}_T \cdot \mathbf{n}}{r[1 - Ma]} \right]_{\ret} d\Gamma.
$$

(3)

Furthermore, $r$ accounts for the vector from the surface to the observer, $Ma$ for the Mach number in the direction of $r$, and $[W]_{\ret}$ states that the terms are evaluated at retarded time. It is noted that the original formulation (3) only incorporates impermeable surfaces. By applying a more general formulation, we incorporate permeable surfaces and, as a result, neglect moving surfaces since the rotating impeller region can be omitted. This allows to cancel out the third part of equation (3), which considers moving surfaces. Furthermore, viscous stresses are neglected since they do not contribute significantly to such technical devices with high Reynolds numbers. Finally, this brings up [35]

$$
\rho' = \frac{1}{4 \pi c_0^2} \nabla \cdot \nabla \cdot \int_{\Omega} \left[ \frac{[L]}{r[1 - Ma]} \right]_{\ret} d\Omega
$$

$$
- \frac{1}{4 \pi c_0^2} \nabla \cdot \int_{\Gamma} \left[ \frac{[P] + \rho \mathbf{v} \otimes \mathbf{v}}{r[1 - Ma]} \cdot \mathbf{n} \right]_{\ret} d\Gamma.
$$

(4)

The FW–H analogy implemented in Star-CCM+ v.12.06 is based on the free-field Green’s function for computing pressure fluctuations at receiver points. For the simulations presented in this paper, the non-convective Dunn Farassat Padula 1A formulation [35] is considered. This formulation is specially designed for impermeable surfaces, which are part of subsonic rotating geometries [35, 36]. For including convective effects, an effective speed of sound incorporating the propagation speed of pressure waves with convective velocity effects [36]

$$
c_0 = \frac{c_0 (1 - Ma^2)}{\sqrt{1 - Ma^2 \sin^2(\theta) + Ma \cos(\theta)}},
$$

(5)

is considered in the FW–H analogy. Thereby, $\theta$ denotes the angle between the convective velocity and the direction of propagation. Based on this implementation, restrictions for in-duct propagation simulation of turbocharger
Consequently, a homogeneous Neumann boundary condition is applied at $\Gamma_{W1}$.

- Wall boundaries not incorporated in the flow simulation at $\Gamma_{W2}$:

For wall boundaries not incorporated in the flow simulation, two possibilities are discussed. Firstly, assuming that aerodynamic and acoustic pressure fluctuation would exist at $\Gamma_{W2}$ brings up the same consideration as for $\Gamma_{W1}$. Secondly, if only acoustic pressure fluctuations remain at $\Gamma_{W2}$ a sound hard wall boundary condition can be applied, resulting in a homogeneous Neumann boundary condition [22]. Either way, a homogeneous Neumann boundary condition is considered for $\Gamma_{W2}$.

- An internal boundary condition $\Gamma_{IF}$:

A perfectly working outlet boundary condition of the flow simulation is assumed. In such cases, all radiating density fluctuations are damped away, and no reflections propagate back into $\Omega_2$. To avoid reflections at the outlet boundary condition, grid stretching or sponge layers can be applied [16]. As a result, $\Gamma_{IF}$ has not to be incorporated into the propagation computation.

- A free propagation boundary condition $\Gamma_F$:

For incorporating free propagation at $\Gamma_F$, a PML technique [37] with an inverse distance damping function in time domain was applied by adding $\Omega_{PML}$ (see Fig. 1).

To obtain the weak formulation of Lighthill's equation (1), we multiply the partial differential equation with the test function $\varphi \in H^1_0$, integrate over the propagation domain $\Omega_p$, and apply integration by parts to the Laplace term of the differential operator and the source term

$$\nabla \rho' \cdot \mathbf{n} = 0, \quad (8)$$

$\nabla \rho' \cdot \mathbf{n} = (\nabla \rho'_{CFD}) \cdot \mathbf{n}, \quad (6)$

with $\rho'_{CFD}$ as density fluctuations derived by the flow simulation.

- Wall boundaries incorporated in the flow simulation $\Gamma_{W1}$:

In the flow simulation, a non-moving and no penetration no-slip boundary $(n_{Wall} = 0)$ is applied at $\Gamma_{W1}$. Using conservation of momentum by neglecting external forces and considering the definition of Lighthill's stress tensor (2) brings up at $\Gamma_{W1}$

$$\nabla \cdot [L] \cdot \mathbf{n} = -\nabla (c_0^2 \rho') \cdot \mathbf{n}. \quad (7)$$

Consequently, a homogeneous Neumann boundary condition

$$\Gamma_{IF,CFD}$$

$\Omega_2$ $\Omega_1$ $\Omega_0$

Flow Simulation

Inlet BC

No-Slip BC

No-Slip BC

Outlet BC

Propagation Simulation

$\Gamma_{W1}$ $\Gamma_{W2}$ $\Gamma_{IF}$ $\Gamma_{W1}$ $\Gamma_{W2}$ $\Omega_{PML}$

$\nabla \cdot [L] \cdot \mathbf{n} = -\nabla (c_0^2 \rho') \cdot \mathbf{n}.$

Figure 1. Computational domains for solving Lighthill’s inhomogeneous wave equation, and the flow simulation depicting all boundaries.
Separating the boundaries according to \( \Gamma_p = \Gamma_{\text{IF,CFD}} \cup \Gamma_{W1} \cup \Gamma_{W2} \cup \Gamma_{F} \) (see Fig. 1) and applying the corresponding boundary conditions brings up

\[
\int_{\Omega_p} \frac{1}{c_0} \frac{\partial^2 \phi}{\partial t^2} c_0^2 \rho' \cdot \nabla \varphi \, d\Omega + \int_{\Omega_p} \nabla (\rho') \cdot \nabla \varphi \, d\Omega
= \int_{\Gamma_{\text{IF,CFD}}} (\nabla \cdot [L]) \cdot \mathbf{n} \varphi \, d\Gamma - \int_{\Omega_p} \nabla \cdot \mathbf{V} \cdot \nabla \varphi \, d\Omega.
\]

(10)

It is noted that the volume source term is only applied for \( \Omega_1 \) since \( \Omega_2 \) is not part of the flow simulation. Now, (2) is plugged into the surface term of (10) while the viscous stress term is neglected, bringing up

\[
\int_{\Omega_p} \frac{1}{c_0} \frac{\partial^2 \phi}{\partial t^2} c_0^2 \rho' \cdot \nabla \varphi \, d\Omega + \int_{\Omega_p} \nabla (\rho') \cdot \nabla \varphi \, d\Omega
= \int_{\Gamma_{\text{IF,CFD}}} (\nabla \cdot \mathbf{V} \otimes \mathbf{V} + \nabla \cdot \mathbf{p}(I)) \cdot \mathbf{n} \varphi \, d\Gamma - \int_{\Omega_p} \nabla \cdot \mathbf{V} \cdot \nabla \varphi \, d\Omega.
\]

(11)

Consequently, the presented approach in its weak formulation, shows clear similarities with the approach of the FW–H analogy (4). However, due to the applied corresponding boundary conditions [38], solid surfaces are considered in (11) by applying homogeneous Neumann boundary conditions but have to be incorporated in the FW–H analogy (4) as source terms.

For applying LH-FE, the hybrid aeroacoustic workflow [39–43] is considered. The workflow can be separated into four different main steps [38]:

1. For computing aeroacoustic source terms, an unsteady flow simulation is performed.
2. The aeroacoustic source terms are interpolated from the flow to a propagation grid while conserving energy [41].
3. For performing the propagation simulation in the time domain, the source term mean part is filtered [44].
4. The propagation simulation is performed by using the open-source multiphysics FEM solver openCFS v.18.03 [45].

### 3 Simulation setups

For establishing all following results, a point of operation was used for which the “whooshing noise” could be verified with an engine test rig (see Tab. 1). The measurements from a cold gas test rig were used to validate simulation results [38, 46].

#### 3.1 Flow simulation

As the first step, an unsteady and compressible flow simulation has been performed. Thereby, an Improved Delayed Detached Eddy Simulation (IDDES) with an underlying \( k-\omega \) SST turbulence model was considered [7]. A segregated flow solver with a hybrid second-order upwind/bounded-central scheme for convection was used. For turning the impeller region, a rigid body rotation was applied. The computation grid utilizes polyhedral cells with refinements at regions with high-velocity gradients, e.g. in the blade tip gap, and prism layers for the boundary layer were used. Considering a grid size of 1.6 mm in the inlet and outlet region and a grid size of 1.3 mm in the impeller region in combination with 10–18 prism layers brought up a grid with 22 417 322 cells. The used time step size corresponds to 1° of rotation of the impeller resulting in \( \Delta t = 1.878 \times 10^{-5} \) s. In addition, for time-stepping a second-order implicit time-stepping scheme was used. The usage of an implicit temporal solver allows simulations of compressible flows with Courant–Friedrichs–Lewy (CFL) numbers [16, 33]

\[
\text{CFL} = \frac{|v| + c) \Delta t}{\Delta x} \geq 1,
\]

(12)

if the mean value is close to \( \text{CFL} \approx 1 \), with \( \Delta x \) as local cell size. For the established grid, an averaged CFL number of 1.284 with a maximal value of \( \text{CFL}_{\text{max}} = 86.39 \) was obtained. Thereby, \( \text{CFL} \leq 1 \) holds for about 3.2%, and \( \text{CFL} > 1 \) for about 96.8% of all cells. As boundary conditions, a mass flow inlet and a pressure outlet were

<table>
<thead>
<tr>
<th>( n ) in 1/min</th>
<th>PR</th>
<th>( m ) in kg/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>88 740</td>
<td>1.7</td>
<td>0.12</td>
</tr>
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### Table 1. Point of operation used for all following measurements and simulations with \( n \) denoting the speed of rotation, \( \text{PR} \) denoting the pressure ratio, and \( m \) denoting the mass flow rate.

![Figure 2. LH-FE setup for the inlet propagation region (\( \Omega_{\text{inlet}} = \Omega_{1,i} \cup \Omega_{2,i} \)) and the outlet propagation region (\( \Omega_{\text{outlet}} = \Omega_{1,o} \cup \Omega_{2,o} \)) with PML regions, and the FW–H setup showing both configurations, the Cropped Geometries and the Full Geometry. Also shown are evaluation points used in Figure 7.](image-url)
considered [16]. Furthermore, the all $y^+$ wall treatment was applied to address wall boundary layers. This approach automatically detects local $y^+$ values and applies the best fitting wall treatment. For this investigation a trade-off between grid size, $y^+$ values, and computational effort was considered, resulting in averaged $y^+$ values of around unity at the blades and overall averaged $y^+/C_{25} = 7.9$ for the entire flow domain due to spatial coarsening in undisturbed regions. For varying $y^+$ values, the all $y^+$ wall treatment delivers adequate results [16, 17]. Furthermore, considering the surface averaged flow velocity $v_F$ at $\Gamma_{IF,CFD}$, as shown in Figure 2., brings up Reynolds numbers of $Re_{in} \approx 137 \times 10^3$ at the inlet, and $Re_{out} \approx 187 \times 10^3$ at the outlet, respectively. For a detailed explanation of the used flow simulation setup, we refer to [38, 47].

In addition, all FW–H simulations were performed parallel to the flow simulation in Star-CCM+ v.12.06.

### 3.2 Propagation simulation

The propagation simulation setup for the LH-FE approach is generally different from the flow simulation setup. For performing a flow simulation, it is essential to resolve length and time scales of turbulent eddies. Consequently, spatial refinements in regions with, e.g. high-velocity gradients have to be applied. In contrast, the propagation grid has to be uniform for adequate wave transportation to prevent numerical reflections in the propagation domain [22]. A common approach is to approximate the wavelength $\lambda$ corresponding with the highest frequency of interest, with $10–20$ finite elements [22, 40–43, 48]. For our computations, the frequency range of $1000 \text{ Hz}$ to $20000 \text{ Hz}$ is of interest, bringing up

$$\lambda_{\text{min}} = \frac{c}{f_{\text{max}}} = \frac{341 \text{ m/s}}{20000 \text{ Hz}} = 1.705 \text{ mm}.$$  

(13)

Based on [40, 41, 43, 48], we use $10$ finite elements per wavelength for spatial discretization, arriving at a mesh resolution of

$$\Delta h = \frac{\lambda_{\text{min}}}{10} = 0.0017 \text{ m} \approx 1.7 \text{ mm}.$$  

(14)

Finally, the propagation grid is assessed by

$$\Delta h \leq \lambda_{\text{min}}.$$  

(15)

Using this propagation grid as target grid, source terms are interpolated applying the least square interpolation approach implemented in Star-CCM+ [49]. The accuracy of this interpolation approach was assessed by interpolating the pressure $p_{\text{CFD}}$ established by the flow simulation onto the propagation grid resulting in $p_p$ and computing a relative error.

![Figure 3. Sum of the absolute values of Lighthill’s source term $\nabla \cdot \nabla \cdot [L]$ and its three main parts of the inlet and outlet region. For processing data of $t = 0.00270453 \text{ s}$, corresponding to 4 turns, and a Hamming window function was considered.](image)
In (16) \( \Omega_p \) denotes the propagation grid and \( \Omega_{CFD} \) the flow grid. Furthermore, for this investigation, \( \Omega_p \) and \( \Omega_{CFD} \) were designed to be the same size and incorporated the inlet region. This investigation showed a relative error of \( \epsilon \approx 8.65\% \) for the whole domain, which was in very good agreement with other interpolation algorithms [38].

Thereby, it is noted that during the interpolation from the flow to the propagation grid, only source terms of \( \Omega_1 \) are considered, neglecting source terms of \( \Omega_0 \). In the next step, the computed Lighthill’s source terms, and its three parts are investigated for the inlet and outlet region of the propagation grid (see Fig. 2). Because of the introduced interface boundary condition \( \Gamma_{CFD} \), the computation of Lighthill’s source term is restricted to parts of the inlet and outlet region (see Fig. 2). As a result, the source term interpolation is just performed for the inlet and outlet region. This allows reducing the amount of exported data significantly. Furthermore, the required simulation time of the LH-FE propagation computation decreases since smaller computation domains can be used. Consequently, the efficiency of the entire hybrid workflow increases, hence enabling to consider more time steps.

4 Results

Subsequently, an extensive source term investigation and results of the flow simulation, the LH-FE, and the FW–H propagation approach are shown and compared.

4.1 Source term investigation

For investigation, the cell-centroid-based source terms of a simulation time of 0.27 ms, which corresponds to four impeller turns, are weighted with the finite element volume \( V_E \) and transformed into the frequency domain while using a Hamming window function. The real and imaginary parts of the transformed source terms of the whole domain are summed up for each frequency, and the magnitude \( S_{[L]}(f) \) is computed [50]

\[
S_{[L]}(f) = \left[ \sum_{i=1}^{N_E} |\Re(V_E^i \nabla \cdot \nabla \cdot [L]^E)|^2 \right]^{1/2} \quad \text{(17)}
\]

Investigating Lighthill’s source term \( \nabla \cdot \nabla \cdot [L] \) (see Fig. 3a) in the inlet region (see Fig. 2) shows that the superposition of the three source term parts brings up a full Lighthill source term with amplitudes larger than the amplitudes of the three single parts. Furthermore, the parts due to the Reynolds stresses and the viscous stresses are small compared to the dominant excess source term. This is interesting since, due to the low Mach number in the inlet region, a dominant Reynolds source term was expected. In the inlet region, an averaged Mach number \( \tilde{Ma} \approx 0.1 \) was computed. As a result, this suggests a significant influence of the phases of the source term parts. Furthermore, since no source terms connected to the blade passing frequency (BPF) move far upstream into the inlet region, no dominant peaks can be seen. Investigating the source
term $\nabla \cdot \nabla \cdot [L]$ in the volume of the outlet region (see Fig. 3b) shows a similar pattern of the three Lighthill source term parts and their superposition. A higher influence of the excess term in the outlet region seems physical, since a higher averaged Mach number $\overline{Ma_{25C2}} = 16$ was computed. However, the large difference between the Reynolds and excess term again suggests a significant influence of the phases of the source term parts. For the outlet region, a dominant peak at the BPF and at the first harmonic of the BPF can be seen. This seems physical because source terms associated with the BPF originate in the impeller region and then travel downstream into the outlet. The slight shift between the actual peak at the BPF is connected to the coarse frequency resolution.

Due to the unexpected behaviour in the inlet, a more detailed investigation of the influence of the phase of Lighthill’s equation (10), we furthermore investigate the two different source term parts, $\nabla \cdot [L] \cdot n$ and $c_0^2 \nabla \rho' \cdot n$ as well as their superposition at $\Gamma_{IF,CFD}$ of the inlet and outlet region.

Hereinafter source terms located at $\Gamma_{IF,CFD}$ are referred to as surface source terms. It is mentioned that $\nabla \cdot [L] \cdot n$ incorporates source term parts that arise at $\Gamma_{IF,CFD}$. The second part $c_0^2 \nabla \rho' \cdot n$ incorporates density fluctuations $\rho'$ which propagate into the domain by $\Gamma_{IF,CFD}$.

Figure 5. Surface weighted Lighthill source terms at the interface surface boundary $\Gamma_{IF,CFD}$ of the inlet and outlet region in the frequency domain of $t = 0.01352265$ s.
of the omitted geometries. For investigating the source term parts at $\Gamma_{\text{IF,CFD}}$, the surface weighted surface source terms are summed up for 1000 Hz $\leq f \leq 20\,000$ Hz

\[ S_{\text{IF,CFD}} = \sum_{j=1}^{n_f} \left( \sum_{i=1}^{n} (a_i^j W_{i,j}) \right), \]

with $W$ as $\nabla \cdot [L] \cdot n$, $c_s^2 \nabla \rho' \cdot n$, and $(\nabla \cdot [L] + c_s^2 \nabla \rho') \cdot n$. Thereby, it could be shown that for $\Gamma_{\text{IF,CFD}}$ for the inlet region, the term $c_s^2 \nabla \rho' \cdot n$ is dominant (see Fig. 5). Furthermore, it could be shown that at the interface, local peaks connected with the BPF appear. This sounds physical since $\Gamma_{\text{IF,CFD}}$ is located close to the impeller, and density fluctuations arising in the impeller propagate a small distance upstream for this point of operation. Investigating surface source terms at $\Gamma_{\text{IF,CFD}}$ of the outlet regions shows that both source term parts, $\nabla \cdot [L] \cdot n$ and $c_s^2 \nabla \rho' \cdot n$ are of a similar order. Furthermore, a clear peak at the BPF can be seen. Density fluctuations connected with the BPF originate directly after the impeller and propagate further downstream. The interface of the outlet region is located downstream of the outlet volute, which is why strong density fluctuations connected with the BPF propagate into the outlet region via $\Gamma_{\text{IF,CFD}}$.

4.2 Propagation results

Finally, the presented Lighthill’s source terms are considered for solving the LH-FE propagation [45]. Therefore, a time step size of $\Delta t_{\text{prop}} = 5 \cdot 10^{-6}$ s was used. Furthermore, the ideal gas law was considered for computing pressure fluctuations $\rho'$ based on the density fluctuations $\rho$ in the isentropic far-field. Therefore, a constant temperature $T$ was taken into account, which is the middle part of the temperature $T$ simulated by the flow solver at the evaluation points MP Inlet and MP Outlet. Furthermore, for smoothly plugging in source terms in the propagation computation a temporal blending function

\[ f_{t,B} = \begin{cases} \cos \left( \frac{\pi}{2\Delta t_{\text{lim}}} (t - t_{\text{lim}}) \right)^2 & t < t_{\text{lim}} \\ 1 & t > t_{\text{lim}} \end{cases} \]

with $t_{\text{lim}} = \Delta t \cdot n$, and $n = 200$ steps was applied to avoid transient effects in the solution [43]. As shown in Figure 2, volume source terms are only available for a part of the considered propagation domains ($\Omega_{f,W}$) of the inlet and outlet region. As the first step, the individual contribution of surface source terms of $\Gamma_{\text{IF,CFD}}$, the volume source terms, and both of those source terms at once are investigated by performing propagation simulations. Thereby, it could be shown that the volume source term is dominant (see Fig. 6) for the outlet region and contributes significantly for the inlet region. The influence of the surface source term, especially in the outlet region, is comparatively small compared to a strong influence in the inlet region since in the outlet region strong source terms appear in the volume as well due to downstream traveling source terms. Furthermore, investigating results of the outlet region, shows that the transversal mode located at $f_{Q,1} = 3672$ Hz is mainly excited by the volume source terms, since they are distributed over a larger region. In contrast, surface source terms located at $\Gamma_{\text{IF,CFD}}$ can not similarly excite transversal modes. In the turbocharger compressor, strong turbulent eddies are generated in the impeller and, for points of operation close to the design point, travel with the flow in the outlet region. For points of operation close to the surge line, noteworthy turbulent eddies, and as a result, noteworthy Lighthill source terms, also travel from the impeller region upstream into the inlet region. Therefore, it is noted that the described influences of surface and volume source terms may vary for different points of operations. E.g., points of operation with stronger backflow throughout the impeller into the

![Figure 6](image-url)
inlet region show different flow characteristics, and the influence of the volume source may further increase. However, the source terms at the interface boundary condition contain not just source terms located exactly at \( \Gamma_{\text{IF,CFD}} \) (by incorporating \( \nabla \cdot [L] \cdot n \)) but also density fluctuations propagating from the impeller into the inlet region (by incorporating \( c_0^2 \rho' \cdot n \)). Consequently, if \( \Gamma_{\text{IF,CFD}} \) is located in a way that no significant Lighthill source terms remain but just density fluctuations, the surface source term will be dominant. Concluding, it can not be said in general which of the source terms, the one located in the volume or at the interface boundary condition, are the most dominant. However, if the fluid is quiescent, no source terms appear and \( c_0^2 \rho' \cdot n \) will be dominant.

For computing FW–H propagation results, two different setups were considered (see Fig. 2). The setup Full Geometry considers all impermeable surfaces of the turbocharger compressor. The setup Cropped Geometry considers all impermeable and permeable surfaces corresponding to the surfaces of the inlet and outlet propagation domain used for LH-FE (see Fig. 2). It is noted that the Cropped Geometry setup neglects all areas with \( Ma > 1 \) appearing in the impeller region, which are included in the Full Geometry.

Investigating the results from the flow simulation, LH-FE, and the FW–H analogy by comparing them with measurements from the cold gas test rig shows that FW–H of the Cropped Geometry significantly underestimates amplitudes for both, the inlet (see Fig. 8a) and outlet (see Fig. 8b) regions. Furthermore, it can be seen that flow simulation results are strongly influenced by numeric dissipation [17, 38]. Especially for the evaluation point of the inlet region, amplitudes for higher frequencies are damped.

The evaluation point of the inlet region is more influenced than the evaluation point of the outlet region because it is further away from the impeller, where most of the source terms are generated. In addition, the differences between results of the flow simulation and the LH-FE of the outlet region are small since the considered source terms of the outlet region propagate far downstream, which is why they
are also influenced by numeric dissipation. It is noted that MP Inlet is located 0.34 m away from the source terms of the impeller region. The distance between MP Outlet and source terms can not easily be assessed. Strong source terms are mainly generated in the impeller region and propagate further downstream. As a result, significant source terms can be found in a large area of the impeller, outlet volute, and even the outlet region, making it challenging to define their distance to evaluation points. Furthermore, this shows that source terms, especially in the outlet region, are also influenced by numeric dissipation.

Furthermore, it can be seen that FW–H for the Full Geometry setup delivers results with amplitudes of similar order than results established using LH-FE for the inlet (see Fig. 8a) and outlet (see Fig. 8b) region, and furthermore both are in good agreement with measurements.

The result of the FW–H analogy for the Full Geometry setup shows that in the inlet region the BPF ($f_{BPF} = 8874$ Hz) and its first harmonic ($f_{BPF,1} = 17 748$ Hz) are resolved. To validate this assumption, pressure fluctuations $p'$ derived from the flow simulations at four different evaluation points located in the inlet region are investigated (see Fig. 7). The four evaluation points are placed at the central axis of the inlet and are located between 0.08 and 2.42 times the duct diameter $D_W$ away from the impeller hub. Consequently, since the considered evaluation point for the FW–H simulations is much further away from the impeller hub, it seems unphysical that the BPF and its first harmonic is that dominant in results established for the inlet region since this can not be seen in measurements. Furthermore, the slight frequency shift from the peak connected with the BPF investigated in Figure 7 is connected with the Doppler effect because the main Lighthill source terms are generated at the impeller blades and move further downstream.

The presented LH-FE approach and the FW–H analogy enable propagating pressure fluctuations based on source terms established by compressible flow simulations. The FW–H analogy is especially efficient in computing pressure fluctuations at discrete receiver points. In contrast, LH-FE efficiently computes pressure fluctuations for the whole propagation domain. As a result, for computing coupled vibro-acoustic simulations, the FW–H analogy would have to be performed multiple times for a high number of
receiver points. For such applications, the LH-FE approach seems more suitable.

4.3 Practical aspects

In a next step, investigate the practical aspects of the presented LH-FE approach. As can be seen in Figure 2, a resonator insert as part of the inlet region is used. This resonator geometry is characterized by the transmission loss (TL), which was investigated in detail in [20]. Let’s now consider the TL of the used resonator insert, which has two maxima, one at $f_{TL, max1} \approx 2600\,\text{Hz}$, and one at $f_{TL, max2} \approx 5400\,\text{Hz}$.

Investigating measurements from the cold gas test rig at MP Inlet 1 clearly shows (see Fig. 9a) significantly smaller amplitudes in areas around the maxima of the TL of the resonator insert. To investigate if the presented LH-FE approach can simulate the influence of the resonator insert in a similar way, source terms of the inlet region are considered.

Since in the inlet region, the surface source terms located at $\Gamma_{IF, CFD}$ are in good agreement with results established by volume and surface source terms for the frequency range of interest (see Fig. 6a), the volume source terms are neglected for the following investigation. Only considering surface source terms allows a faster computation of the LH-FE approach. Surface source terms of $\Gamma_{IF, CFD}$ of one flow simulation are interpolated onto two different propagation grids, one including and one neglecting the resonator insert. Performing propagation simulations for those two different propagation setups also clearly show the influence of the resonator insert (see Fig. 9b). Comparing the results established by the cold gas test rig measurement (see Fig. 9a) and by the introduced LH-FE approach (see Fig. 9b) shows a similar influence of the resonator insert.

FW–H’s analogy is not suitable for assessing different in-duct resonators and sound absorbers since source terms at impermeable surfaces are fundamental. In contrast, the LH-FE approach enables to neglect all impermeable surfaces and, as a result, is well suited for assessing different geometries based on one flow simulation.

Performing an enhanced flow simulation as presented, establishing converged results, and exporting source terms of at least 20 impeller turns took more than 3 weeks of extensive computations on the Vienna Scientific Cluster [51], using 2 nodes with 24 physical cores (see Tab. 2). Consequently, computing different flow simulations to investigate the influence of resonator inserts will take a lot of time and will still be influenced by numeric dissipation. However, performing a propagation simulation based on the LH-FE approach took about 10 h to compute (see Tab. 2) using 8 cores on our RK3 compute server (see Tab. 4). A detailed representation of the required computing time of the individual steps can be seen in Table 3. Consequently, the presented LH-FE approach gives a time advantage of about 50 times compared to performing an additional flow simulation.

5 Conclusions

A new FEM approach of Lighthill’s wave equation for compressible flow simulations (LH-FE) incorporating interface boundary conditions for omitting geometry parts is investigated and compared with FW–H for turbocharger acoustic simulations. Analyzing Lighthill’s aeroacoustic source terms in the inlet region, it could be shown that the volume source terms show no dominant influence at the BPF mechanism. In contrast, in the outlet region, the most dominant volume source terms are at the BPF. Furthermore, it is shown that Lighthill’s source terms in the volume can be phased out. Depending on the flow simulation, this can lead to canceling out of different Lighthill’s source term parts (Reynolds stress part, excess part, viscous stress part). Furthermore, it could be shown that depending on the location of the interface boundary condition, the influence of source terms of the volume or the surfaces varies substantially. Directly comparing results obtained by using the LH-FE approach and the FW–H analogy shows that for both propagation regions, the inlet and outlet region, the LH-FE approach is superior. However, if FW–H is applied to the whole turbocharger geometry, results are similar. In addition, it could be demonstrated that both sound propagation approaches are not influenced by numeric dissipation such as flow simulation results. However, based on the hybrid aeroacoustic workflow used for the LH-FE approach back-coupling is neglected. Furthermore, the approach requires a fine flow grid solution for the source term regions of the cropped geometries. Consequently, flow grids tend to consist of more cells than usually. Finally, the influence of a resonator insert was taken into account by considering its transmission loss and investigating its influence in measurements and simulations using the LH-FE approach. It could be shown that the LH-FE approach can represent the influence of the resonator insert by just using source terms of one flow simulation. Consequently,
the presented LH-FE approach is about 50 times faster for assessing the influence of resonators compared with flow simulations.

Conflict of interest

The authors declare no conflict of interest.

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